Core Mathematics

Unit 4

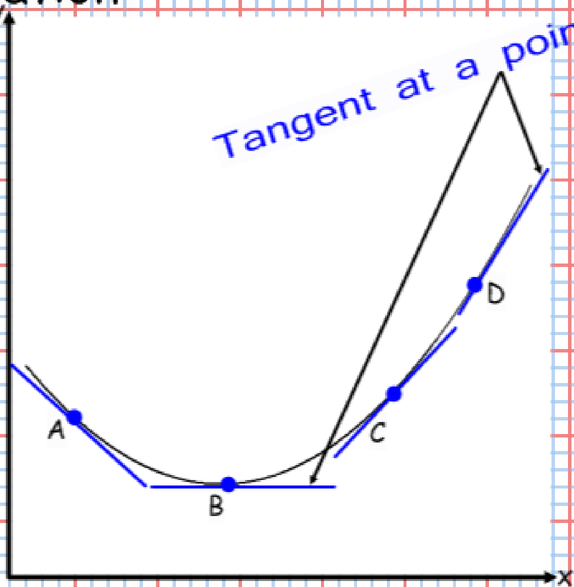
TUTORIAL Lesson Book

name of student...........................

Differentiation

$y = x^n$

Power Function



$f'(x) = \frac{dy}{dx} = nx^{n-1}$

first differential

$f''(x) = \frac{d^2y}{dx^2} = \frac{dy}{dx}\left[f'(x)\right]$

second differential

Example

$y = 5x^7$

$\frac{dy}{dx} =$ 	$\frac{d^2y}{dx^2} =$

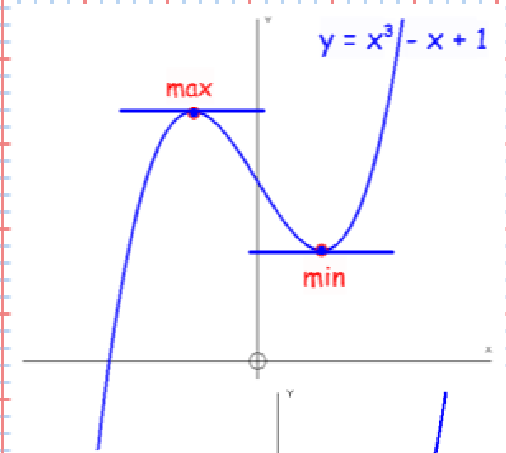
stationary point

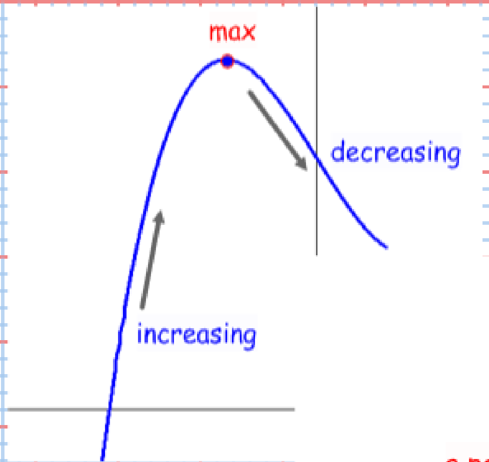
is any point on a curve where the gradient is 0

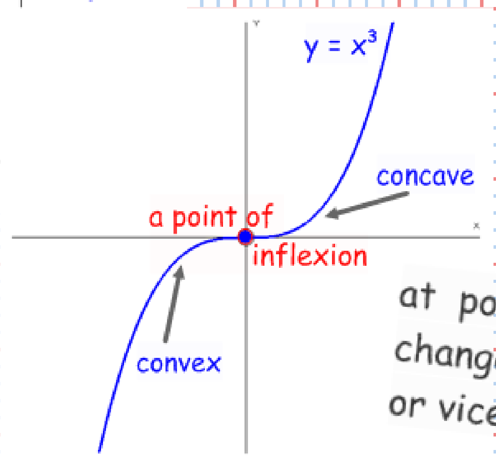
$f'(x) = \frac{dy}{dx} = 0$

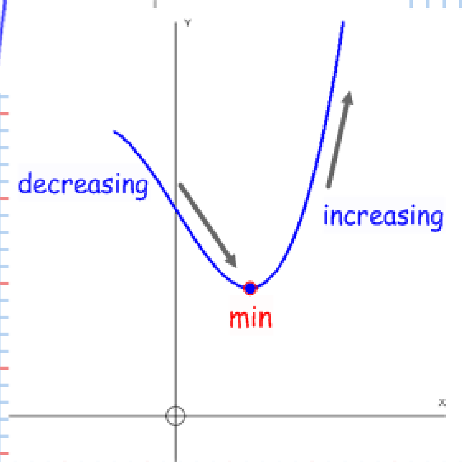
3 types of stationary point

- maximum turning point
- minimum turning point
- a point of inflexion



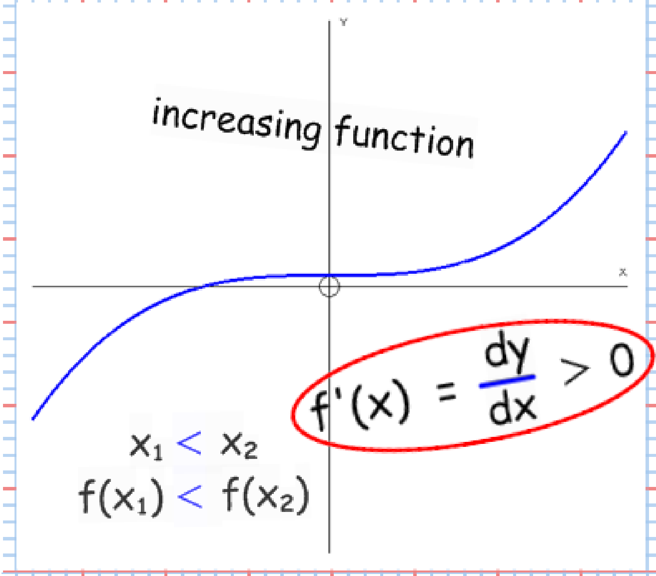


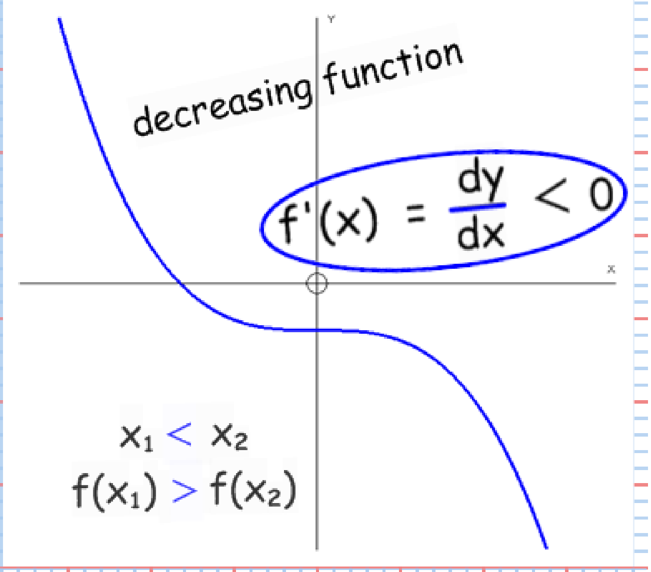




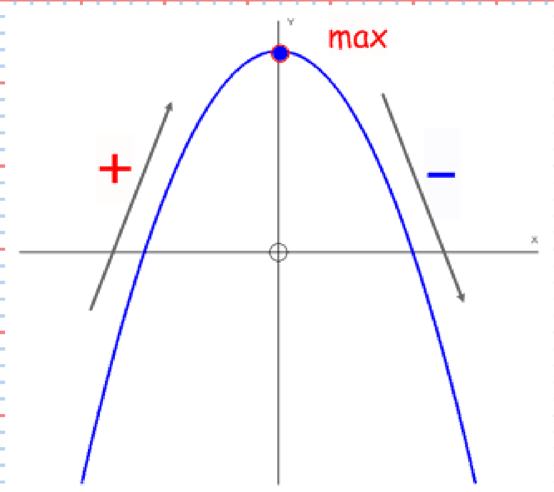
at points of inflexion a curve changes from concave to convex or vice versa

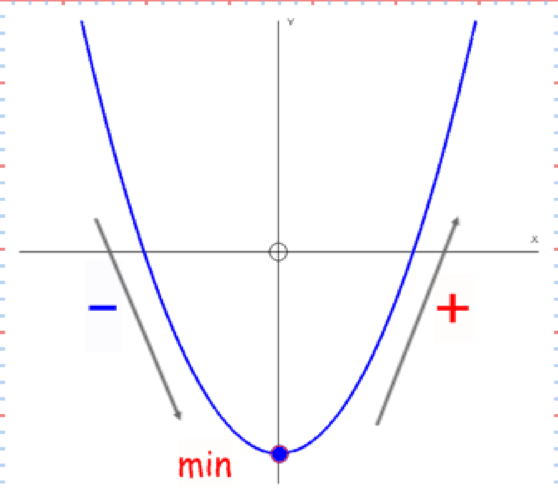
increasing and decreasing functions





Stationary Points and Tangents



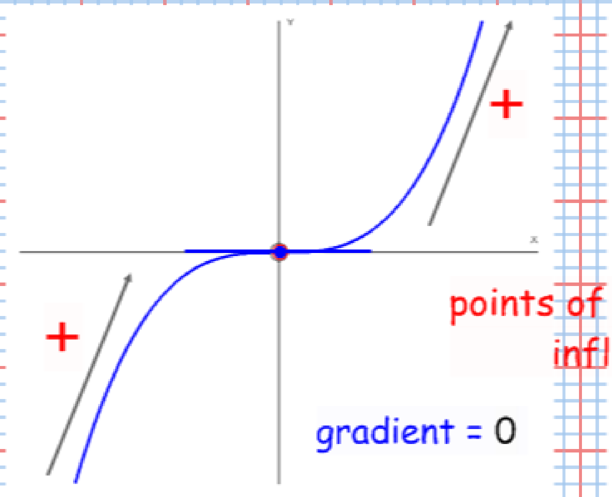


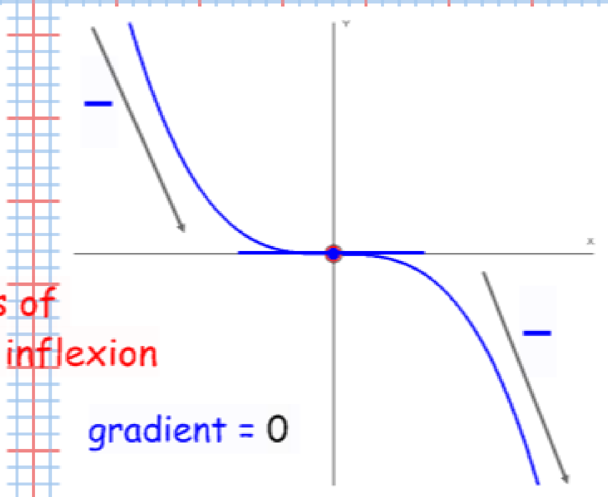
gradient = 0 at max

gradient changes from + to –

gradient = 0 at min

gradient changes from – to +





gradient on either side has the same sign

The Use of Second Differential Function (Second Derivative Function)

- $\frac{dy}{dx} = 0 \quad \frac{d^2y}{dx^2} < 0 \quad \Rightarrow \quad f(x) = max$
- $\frac{dy}{dx} = 0 \quad \frac{d^2y}{dx^2} > 0 \quad \Rightarrow \quad f(x) = min$
- $\frac{dy}{dx} = 0 \quad \frac{d^2y}{dx^2} = 0 \quad \frac{d^3y}{dx^3} \neq 0 \quad \Rightarrow \quad f(x) =$ the point of inflexion
- $\frac{dy}{dx} = 0 \quad \frac{d^2y}{dx^2} = 0 \quad \frac{d^3y}{dx^3} = 0 \quad \Rightarrow \quad f(x) =$ either max or min

$\frac{d^3y}{dx^3}$ third differential

Nature of Stationary Point.doc

Example The Nature of Stationary Points

a) Find and classify any stationary points on the curve $y = x^4 + x^3$

$$\frac{dy}{dx} = 4x^3 + 3x^2 = 0 \quad \Rightarrow \quad x^2(4x + 3) = 0$$

$$\Rightarrow \quad x = 0 \text{ or } x = -\frac{3}{4} \quad \Rightarrow \quad y = 0 \text{ or } y = -\frac{27}{256}$$

the stationary points are $(0,0)$ and $\left(-\frac{3}{4}, -\frac{27}{256}\right)$

$$\frac{d^2y}{dx^2} = 12x^2 + 6x \quad \text{when } x = -\frac{3}{4} \quad \frac{d^2y}{dx^2} = \frac{9}{4} > 0$$

$$\Rightarrow \left(-\frac{3}{4}, -\frac{27}{256}\right) \text{ is the minimum point}$$

when $x = 0$ $\frac{d^2y}{dx^2} = 0$, so we need to investigate further

$$\frac{d^3y}{dx^3} = 24x + 6$$

when $x = 0$ $\frac{d^3y}{dx^3} = 6 \neq 0$

x	$-\frac{1}{4}$	0	$\frac{1}{4}$
$\frac{dy}{dx}$	$+$ $\frac{1}{8}$	0	$+$ $\frac{1}{4}$

$\Rightarrow$ $(0,0)$ is the point of inflexion

b) Sketch the curve $y = x^4 + x^3$

when $y = 0$

$x^4 + x^3 = x^3(x + 1) = 0$

$x = 0$

or

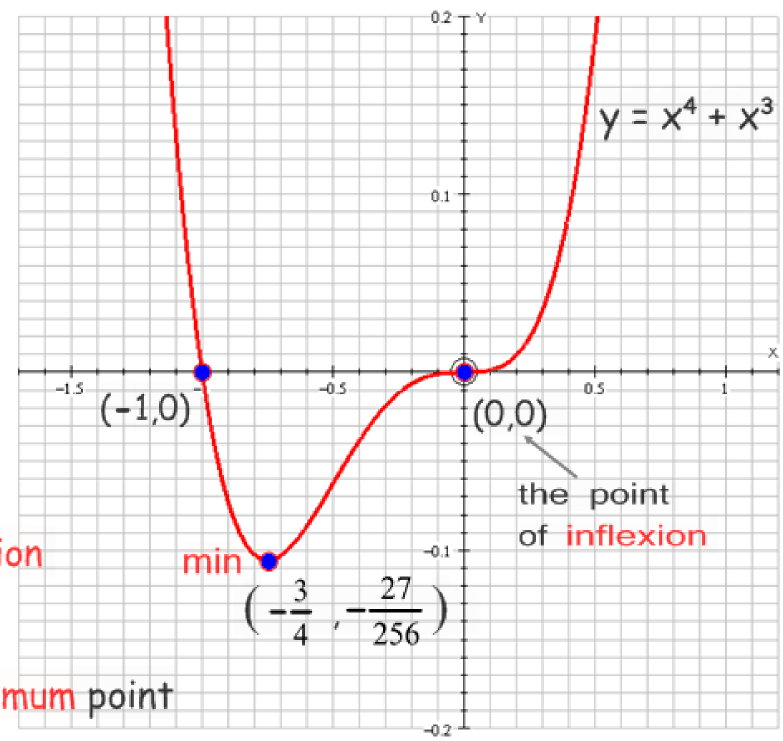
$x = -1$

$\Rightarrow$

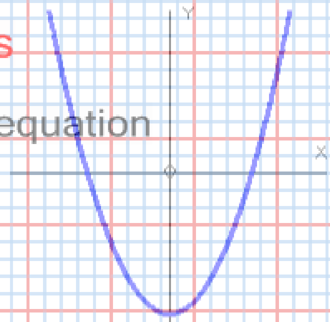
(0,0) and (-1,0)
are the zero points

(0,0) is the point of inflexion

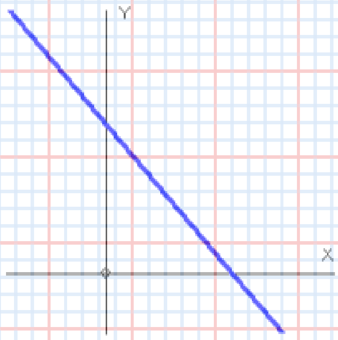
$\left(-\frac{3}{4}, -\frac{27}{256}\right)$ is the minimum point



General Guidance for Sketching Curves



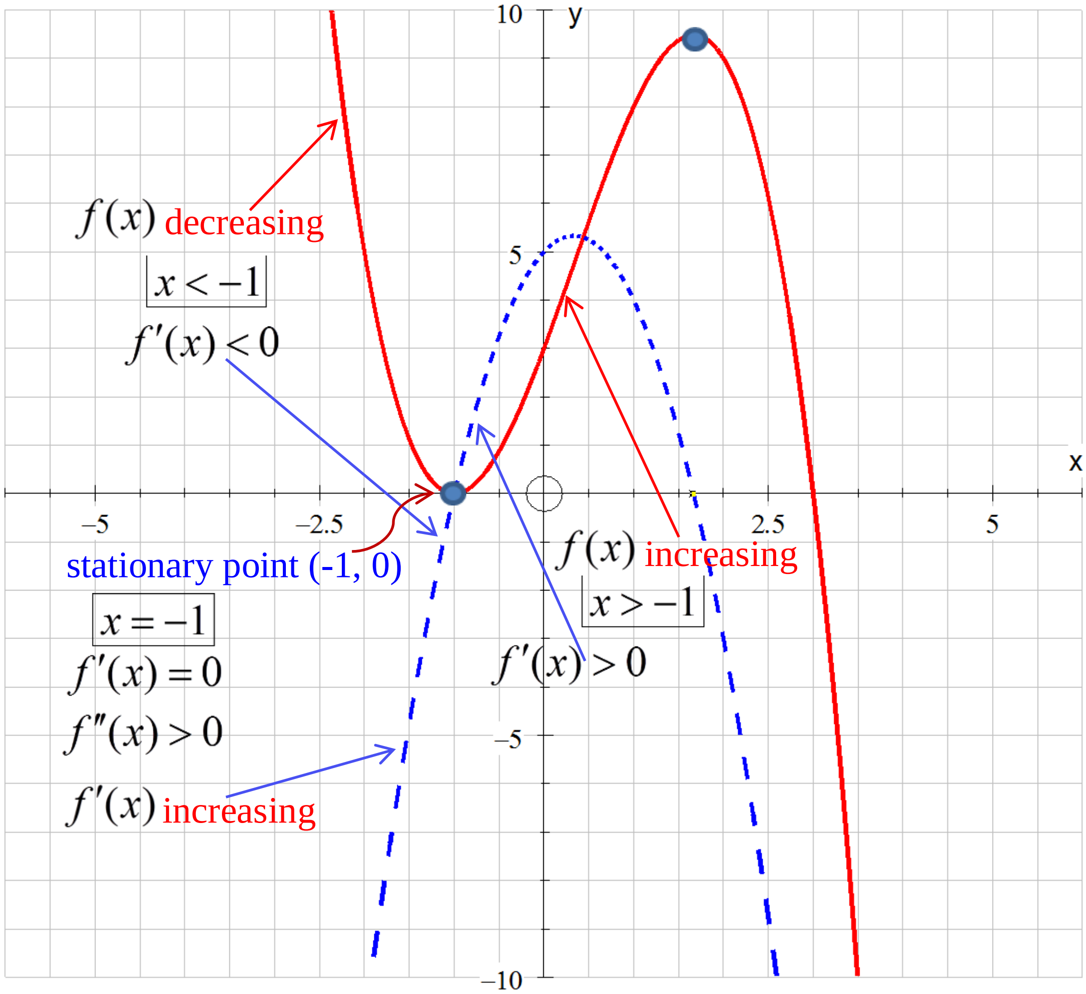
- recognise the shape of the graph from the equation
- find where the curve cuts the X-axis
- find where the curve cuts the Y-axis
- investigate the type and position of any stationary points
- investigate the behaviour when X or Y are endlessly large both positive and negative



Nature of Stationary Point

Function $f(x) = -x^3 + x^2 + 5x + 3$

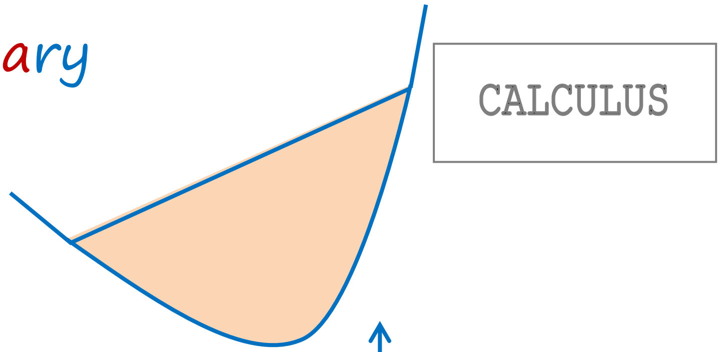
Differential Function $f'(x) = -3x^2 + 2x + 5$



Second Differential Function $f''(x) = -6x + 2$ is **positive** at $f''(x) > 0$

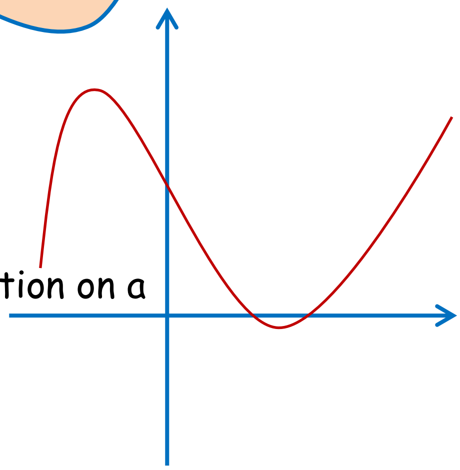
Differential $f'(x)$ is **increasing** when going through this point (that is from negative to positive) $\Rightarrow$ the Function $f'(x) < 0$ is changing from decreasing to increasing $\Rightarrow$ stationary point (-1, 0) is the local minimum

Vocabulary



Area

– the amount of space inside a shape.



Curve

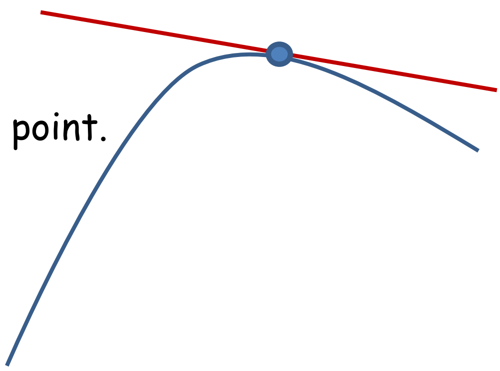
– a bending line, without angles / the graph of a function on a coordinate plane.

Calculus

– comes from Latin meaning 'small stone', because it is like understanding something by looking at small pieces.

Differential Calculus cuts functions into small pieces to find how it changes.

Integral Calculus joins the small pieces together to find the areas under curves.



Tangent

– a straight line that "just touches" the curve at a point.

Differentiate

– find the differential of a function.

Example: $\frac{dy}{dx}(3x^2 - 2x + 5) = 6x - 2$.

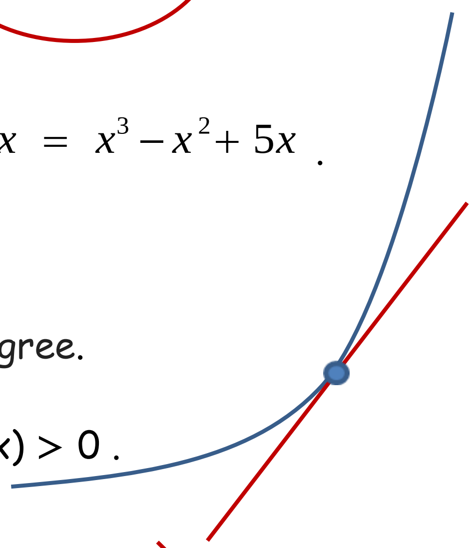
$$\frac{dy}{dx}$$

$$\int f(x)\,\mathrm{d}x$$

Integrate

– find the integral of a function.

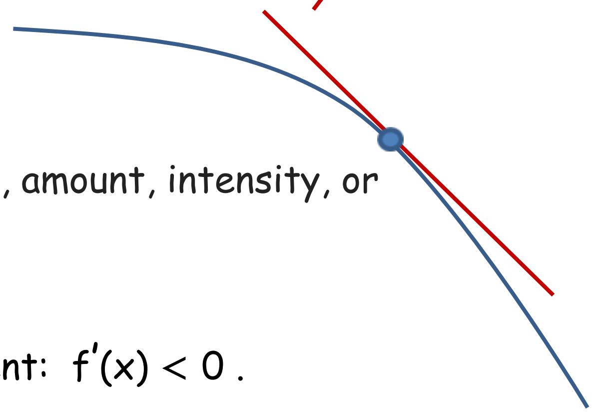
Example: $\int \left(3x^2 - 2x + 5\right)\, dx = \frac{3x^3}{3} - \frac{2x^2}{2} + 5x = x^3 - x^2 + 5x$.



Increasing

– become or make greater in size, amount, or degree.

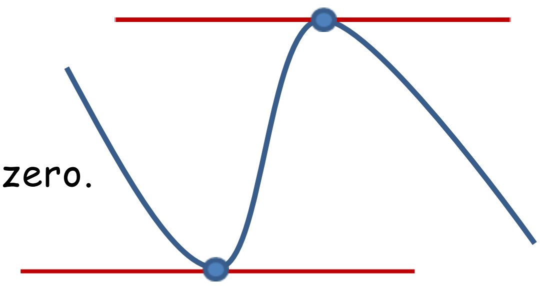
Increasing Function has a positive gradient: $f'(x) > 0$.



Decreasing

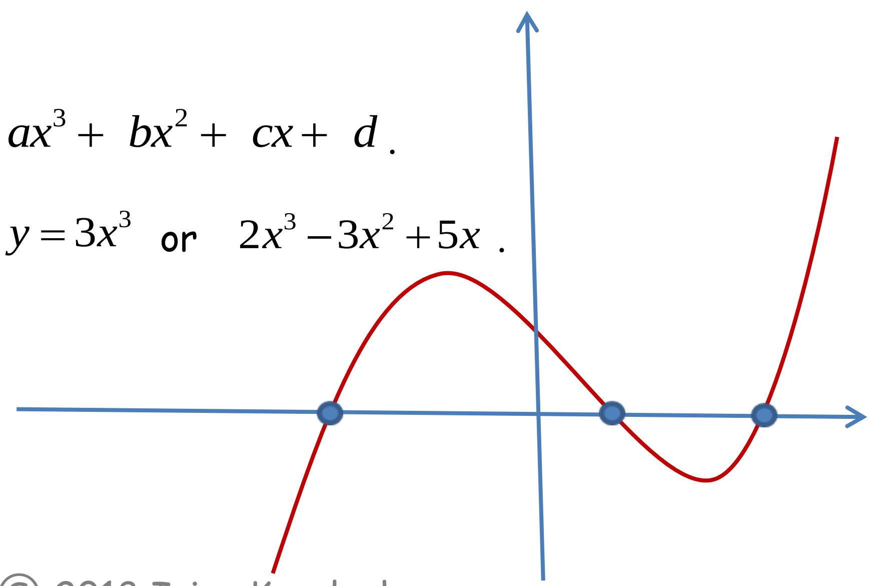
– make or become smaller or fewer in size, amount, intensity, or degree.

Decreasing Function has a negative gradient: $f'(x) < 0$.



Stationary Point

– any point on a curve where gradient is zero.



Cubic Function

– a function in the form $y = ax^3 + bx^2 + cx + d$.

Example: $y = 2x^3 - 8$ or $y = 3x^3$ or $2x^3 - 3x^2 + 5x$.